Wechselstromkreise

mit Induktivität

$$U_{ind} = -L \cdot \dot{I}$$

$$U = U_0 \cos\omega t$$

$$I = I_0 \sin\omega t$$

$$\dot{I} = +I_0\,\omega \cos\omega t$$

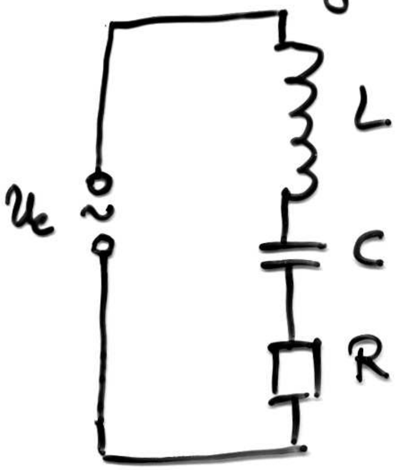
mit Kapazität

$$U = \frac{Q}{C} \qquad \frac{dU}{dt} = \frac{1}{C}\cdot\frac{dQ}{dt} = \frac{1}{C}\cdot I$$

$$Z = \frac{U}{I} = e^{-i\pi/2}\,\frac{U_0}{I_0} = -i\,\frac{U_0}{I_0}$$

$$= -i\,\frac{1}{\omega C} = \frac{1}{i\omega C}$$

allgemeiner Fall



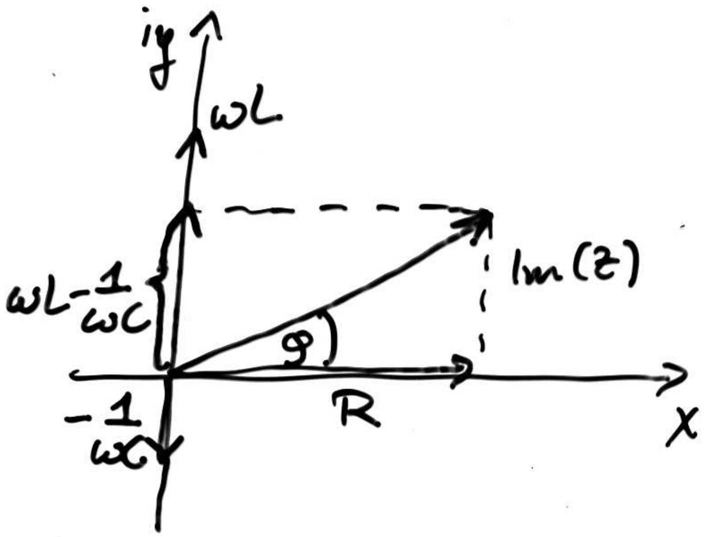
$$U_e = L\,\frac{dI}{dt} + \frac{Q}{C} + I\cdot R$$

$$\frac{dU_e}{dt} = L\cdot\frac{d^2I}{dt^2} + \frac{1}{C}\,I + R\cdot\frac{dI}{dt}$$

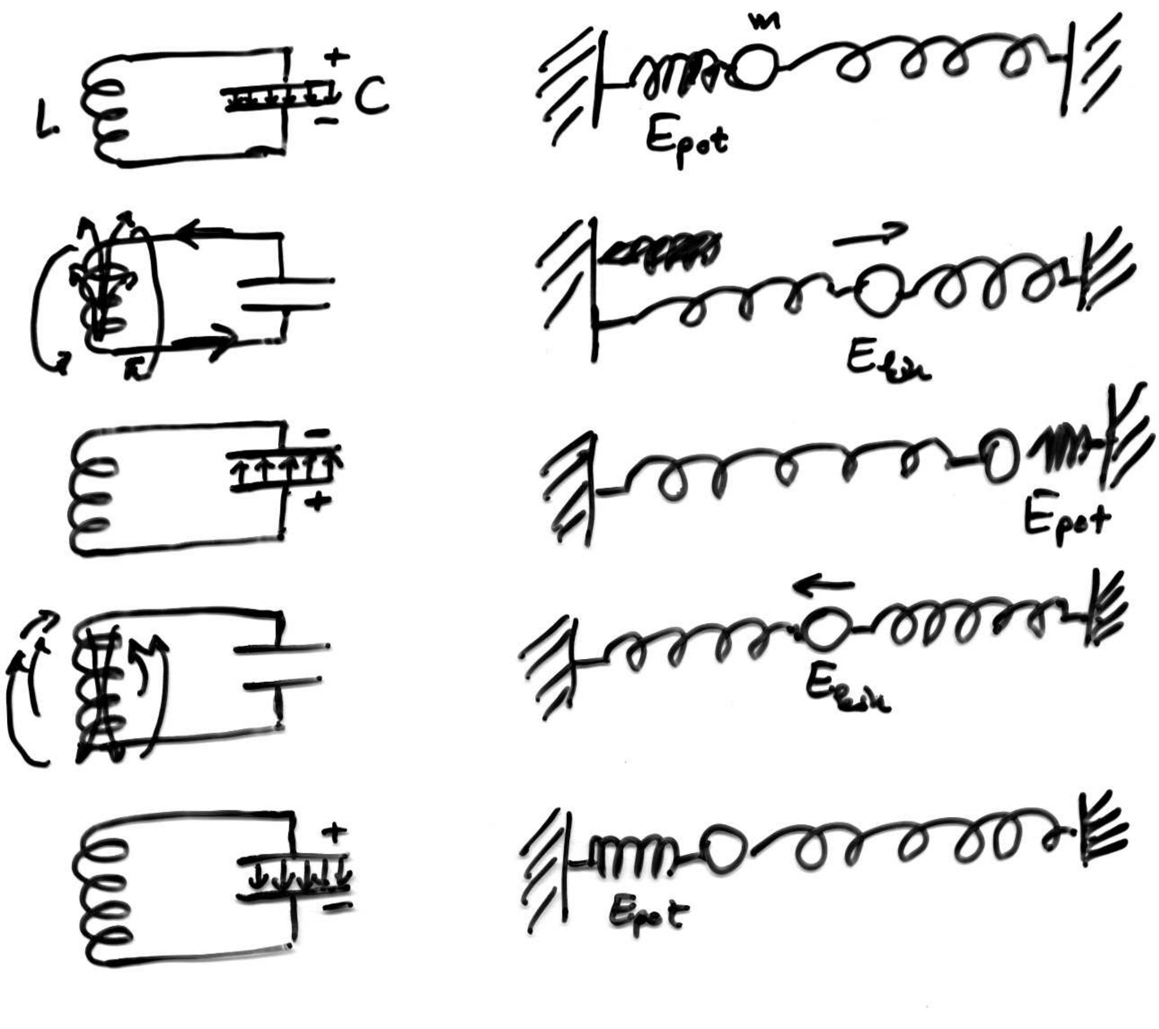
$$U_e = U_0\,e^{i\omega t} \qquad I = I_0\,e^{i(\omega t - \varphi)}$$

$$\boxed{Z = \frac{U}{I} = R + i\left(\omega L - \frac{1}{\omega C}\right)}$$

$|Z|$ Impedanz



Elektromagnetische Schwingungen



$$L \cdot \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{1}{C} I = 0$$

$$I = A e^{\lambda t}$$

$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0$$

$$\lambda_{1/2} = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$= -\alpha \pm \beta$$

$$I = A_1 e^{-(\alpha-\beta)t} + A_2 e^{-(\alpha+\beta)t}$$

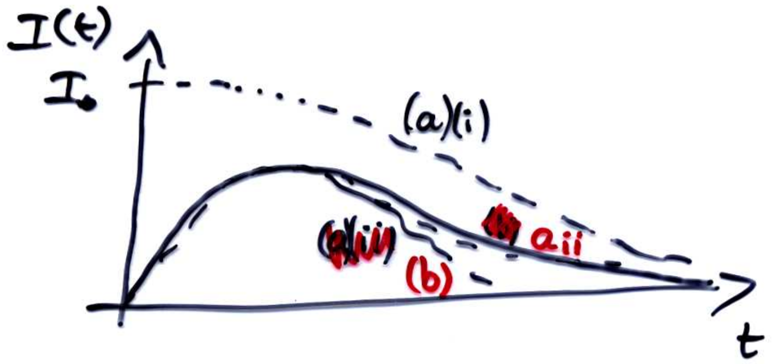
a) Kriechfall

$$\frac{R^2}{4L^2} > \frac{1}{LC} \Rightarrow \beta \text{ reell}$$

(i) $I(t) = I_0 e^{-\alpha t}\left[\cosh(\beta t) + \frac{\alpha}{\beta}\sinh(\beta t)\right]$

für $I(0) = I_0 \quad \dot{I}_0 = 0$

(ii) $I(t) = \frac{\dot{I}_0}{\beta} \cdot e^{-\alpha t} \sinh(\beta t)$ für $I_0 = 0$ und $\dot{I}_0 \neq 0$



b) Aperiodischer Grenzfall

$\beta = 0 \qquad \frac{R^2}{4L^2} = \frac{1}{LC}$

$I(t) = e^{-\alpha t} \, (I_0 + A_3 t)$

für $I_0 = 0 \qquad I(t) = \dot{I}_0 \cdot t \, e^{-\alpha t}$

c) Gedämpfte Schwingung

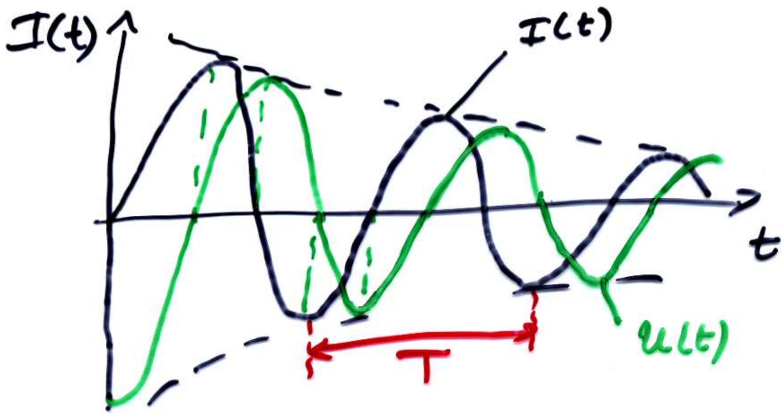
$\frac{R^2}{4L^2} < \frac{1}{LC} \qquad R^2 < 4L/C$

$I(t) = e^{-\alpha t} \left[A_1 e^{i\omega t} + A_2 e^{-i\omega t} \right]$

$\beta = i\omega$

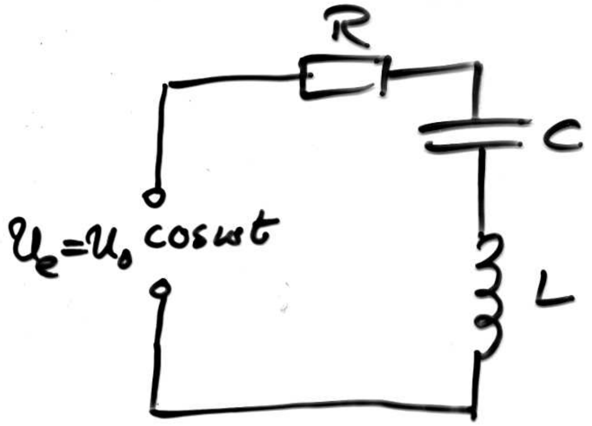
$I(t) = 2 \, |A| \, e^{-\alpha t} \cos(\omega t + \varphi)$

Anfangsbedingungen: A, φ



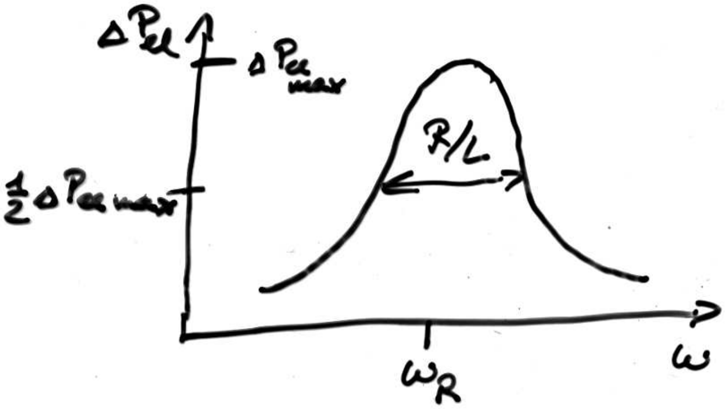
$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad T = \frac{2\pi}{\omega}$

Erzwungene Schwingungen

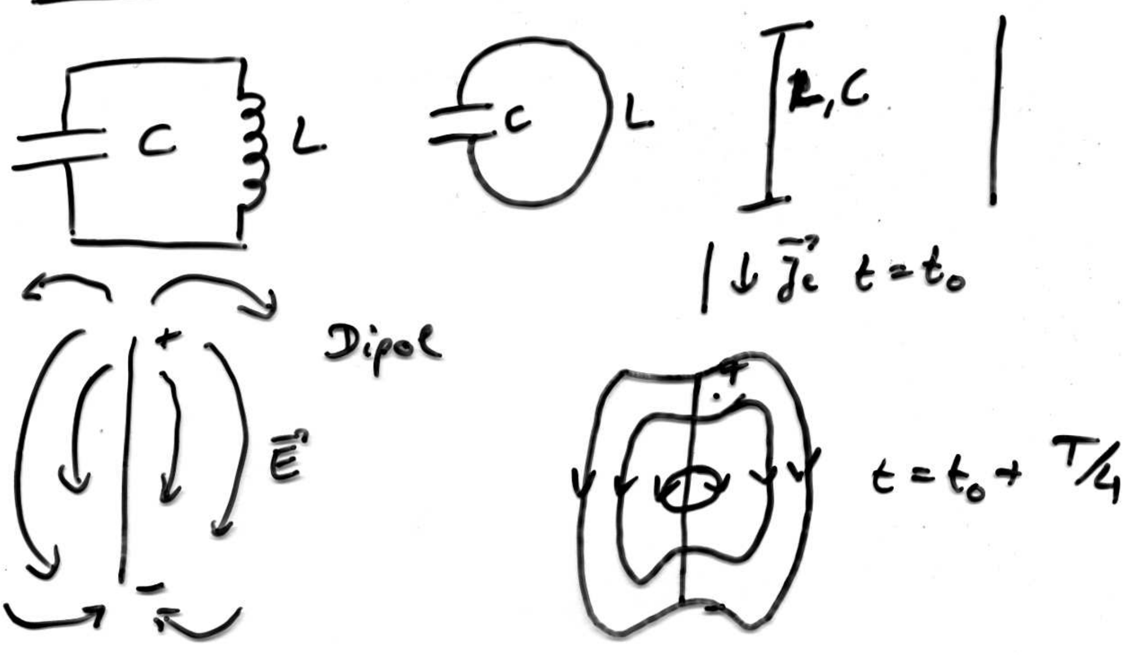


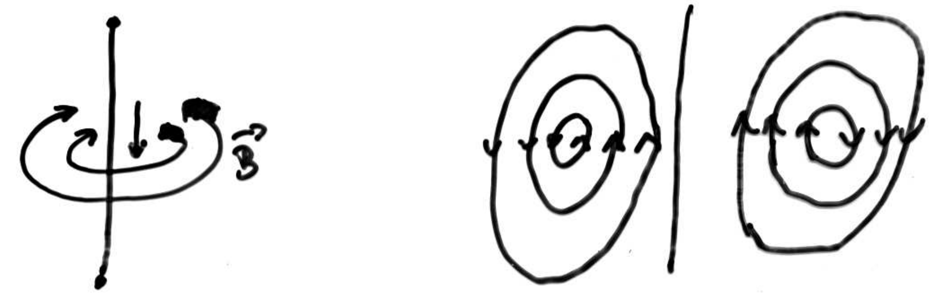
$$L \cdot \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{1}{C} I = \frac{dU_e}{dt}$$

$$\omega_R = \left(\frac{1}{LC} - \frac{R^2}{4L^2} \right)^{1/2}$$

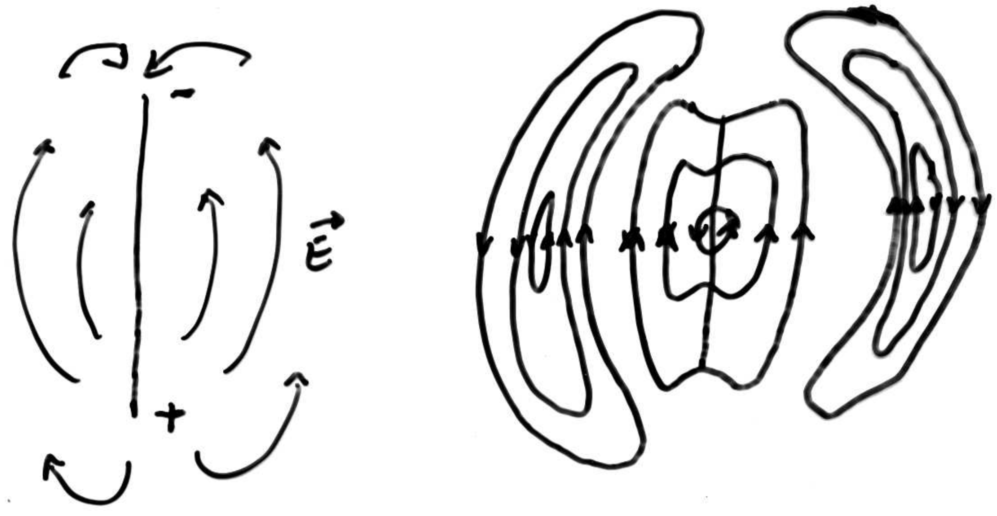


Hertzscher Dipol





$t = t_0 + T/2$



$t = t_0 + \frac{3}{4} T$

$t = t_0 + T$

Abstrahlung des schwingenden Dipols

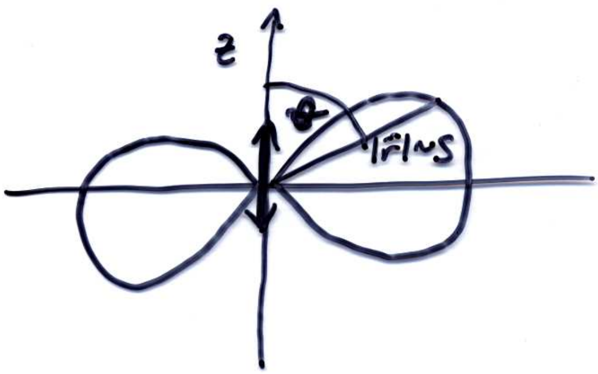
$$w_{em} = \frac{W}{V} = \frac{1}{2} \varepsilon_0 \left(E^2 + c^2 B^2 \right) = \frac{1}{2} \varepsilon_0 E_0^2$$
$$= \varepsilon_0 E_1^2$$

Energie, die pro Zeiteinheit durch Flächeneinheit transportiert wird

$V \rightarrow A \cdot \Delta l \qquad \frac{\Delta l}{\Delta t} = c$

$$S = \varepsilon_0 \, c \, E_1^2 = \frac{\varepsilon_0 E_1^2 V}{A \cdot \Delta t}$$

Fläche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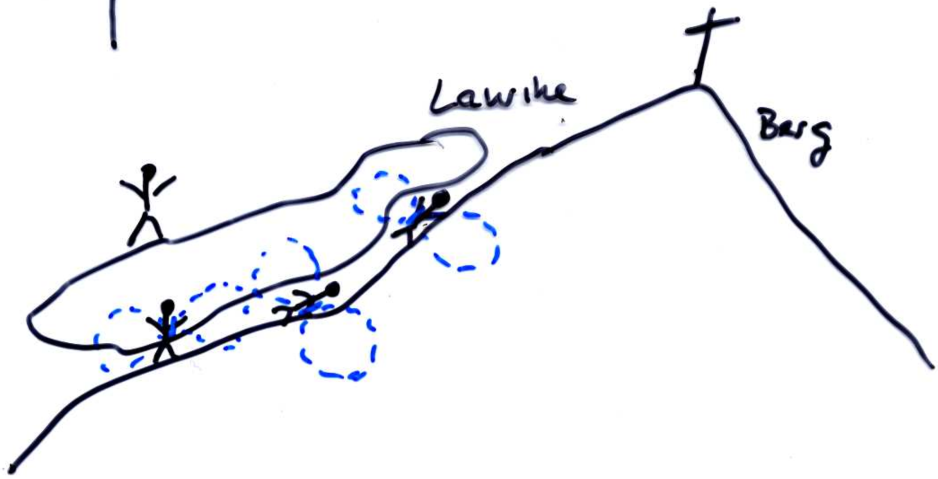
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