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Tutorium 19

Aufgabe 1

a)

$$\begin{aligned}
\Phi &= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \\
&= \int_A \frac{1}{4\pi\varepsilon_0}\frac{\mathsf{d}Q}{R} \qquad \mathsf{d}Q = \sigma\mathsf{d}A \;\; R = \sqrt{r^2+z^2} \\
&= \int_0^{2\pi}\int_0^a \frac{\sigma}{4\pi\varepsilon_0}\frac{1}{\sqrt{r^2+z^2}} r\mathsf{d}\varphi\mathsf{d}r \\
&= \frac{\sigma}{2\varepsilon_0}\int_0^a \frac{r}{\sqrt{r^2+z^2}}\mathsf{d}\varphi \\
&= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{r^2+z^2}\right]_0^a = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2}-z\right)
\end{aligned}$$

b)

$$\begin{aligned}
\vec{E} &= -\mathrm{grad}\Phi \\
\Rightarrow \Phi(z) \Rightarrow \vec{E} &= -\frac{\partial \Phi}{\partial z}
\end{aligned}$$

$$\begin{aligned}
\Phi(z) &= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2}-z\right) \\
E_z &= -\frac{\sigma}{2\varepsilon_0}\left(\frac{2z}{2\sqrt{a^2+z^2}}-1\right) = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{a^2+z^2}}\right) \\
\vec{E} &= E_z\hat{e}_z
\end{aligned}$$

Scheint nicht zu gehen:

$$\begin{aligned}
E &= \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^a \frac{r}{r^2+z^2}\mathsf{d}r\mathsf{d}\varphi \\
&= \frac{\sigma}{\pi\varepsilon_0 z^2}\int_0^a \frac{r}{1+\frac{r^2}{z^2}}\mathsf{d}r \\
&= \ldots
\end{aligned}$$

c)

$$z \gg a$$

$$\begin{aligned}\Phi &= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2}-z\right) = \frac{\sigma}{2\varepsilon_0}z\left(\sqrt{1+\frac{a^2}{z^2}}-1\right) \\ &= \frac{\sigma}{2\varepsilon_0}z\left(1+\frac{1}{2}\frac{a^2}{z^2}-1\right) = \frac{\sigma}{2\varepsilon_0}\frac{a^2}{z} \\ &= \frac{Q}{4\pi\varepsilon_0}z \qquad \text{mit } Q = \frac{\sigma}{A} \Leftrightarrow \sigma = \frac{Q}{\pi a^2}\end{aligned}$$

Für E

$$\frac{1}{\sqrt{a^2+z^2}} = \frac{1}{a}\frac{1}{\sqrt{1+\frac{z^2}{a^2}}} = \frac{1}{a}\left(a-\frac{1}{2}\frac{z^2}{a^2}\right) \qquad \text{mit } (1+a)^\alpha \sim 1+\alpha a+\alpha(\alpha-1)\frac{a^2}{2}$$

$$E_z = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{a}\left(1-\frac{1}{2}\frac{z^2}{a^2}\right)\right) \longrightarrow \frac{\sigma}{2\varepsilon_0}$$

Aufgabe 2

$$\begin{aligned}U(t) &= U_0 \sin \omega t \\ I(t) &= I_0 \sin \omega t\end{aligned}$$

$$\begin{aligned}P &= UI \\ \bar{P} &= \frac{1}{T}\int_0^T U_0 I_0 \sin^2 \omega t \,\mathsf{d}t \\ &= \frac{1}{T}\left[-\frac{U_0 I_0}{2}t - \frac{1}{2\omega}\sin 2\omega t\right]_0^T \\ &= \frac{1}{2}U_0 I_0 \overset{!}{=} U_{Eff} I_{Eff}\end{aligned}$$

$$\begin{aligned}\frac{1}{2}U_0 I_0 &= \frac{1}{2}U_0\frac{U_0}{R} = \frac{U_{eff}^2}{R} \\ \Rightarrow U_0 &= U_{eff}\sqrt{2} \\ \Rightarrow I_0 &= I_{eff}\sqrt{2}\end{aligned}$$

b)

$$U_e = L\dot{I} + IR$$
$$\frac{\mathsf{d}U}{\mathsf{d}t} = L\frac{\mathsf{d}^2 I}{\mathsf{d}t} + \frac{\mathsf{d}I}{\mathsf{d}t}R$$
$$U_e = U_0 \mathrm{e}^{i\omega t}$$
$$I = I_0 \mathrm{e}^{i(\omega t + \varphi)}$$
$$(\text{mit } \tan\varphi = \frac{U_L}{I_R} = \frac{I_0 \omega L}{I_0 \omega R} = \frac{\omega L}{R})$$

$$Z = \frac{U_0}{I_0} = i\omega L + R$$
$$|Z| = \sqrt{\omega^2 L^2 + R^2}$$

Aufgabe 3

a)

$$\vec{m}_m = I\vec{A}$$

$$|A| = \frac{1}{2}a\frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$$

$$\Rightarrow \vec{m}_m = \frac{\sqrt{3}}{4}a^2 I \cdot \hat{e}_z$$

b)

$$\vec{F} = \vec{m}_m \mathrm{grad}\vec{B}$$
$$\vec{F}_m = \vec{0}$$

$$\vec{M} = \underbrace{\vec{m}_m}_{\hat{e}_z} \times \underbrace{\vec{B}}_{\hat{e}_z} = \vec{0}$$

c)

$$\vec{F}_m = 0$$
$$\vec{M} = \underbrace{\vec{m}_m}_{\hat{e}_z} \times \underbrace{\vec{B}}_{\hat{e}_x} = a^2 \frac{\sqrt{3}}{4} IB \cdot \hat{e}_y$$

d)

$$W = -\vec{m}_m \cdot \vec{B} \Rightarrow \Delta W = 2\mu_0 B$$

Aufgabe 3

$$\begin{aligned} U_0 &= U_{R_1} + U_L \\ U_L &= L\dot{I} = U_R \\ I &= I_1 - \frac{U_{R_2}}{R_2} = \frac{U_{R_1}}{R_1} - \frac{U_{R_2}}{R_2} = \frac{U_0 - U_L}{R_1} - \frac{U_2}{R_2} \\ &= \frac{U_0}{R} - \dot{I}L \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}_{\frac{R_1+R_2}{R_1 R_2}} \end{aligned}$$

$$\begin{aligned} I(t) &= I_{max}\left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right) \\ \Rightarrow I_{max} &= \frac{U_0}{R_1} \\ \Rightarrow \tau &= L\frac{R_1 + R_2}{R_1 R_2} \end{aligned}$$

b)

$$I_1 = \frac{U_0 - L\dot{I}}{R_1} = \frac{U_0}{R_1}\left(1 - \frac{R_2}{R_1 - R_2}\mathrm{e}^{-\frac{t}{\tau}}\right)$$

$$\begin{aligned} t = 0 : & I_1 = \frac{U_0}{R_1 + R_2} \\ t \to \infty : & I_1 = \frac{U_0}{R_1} \end{aligned}$$

c)

$$\begin{aligned} E &= \frac{1}{2} L I^2 \\ &= \frac{1}{2} L I_{max}^2 \left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right)^2 \end{aligned}$$